or both a and $b>\$1$. Then as x is an integer, as $a>\$1$, and as B paid but \$6 for the first month's service, $x \gtreqless 5$. Similarly, as y is an integer, $b>\$1$, and as C paid but \$7.20 for the second month's service $(y+1) \gtreqless 7$, or $y \gtreqless 6$. Hence as $n=3+x+y$, $n \gtreqless 14$. But as $c=m/(n+6)=1$, $m=(n+6)$, whence $m \gtreqless \$20$. But as not less than 5 horses were placed in the pasture the first month, $m= \gtreqless 11$.

B had x horses in the pasture the first month, the rate then being m/n, or $m/(m-6)$, and paid \$6 for the service. Hence, $xm/(m-6)=36$, or reducing and transposing, $(6-x)m=36$, but as $(6-x)$=integer, and $11 \gtreqless m \gtreqless 20$ (remembering that C had some horses in the pasture the first month, and that $n=m-6$). $6-x=2$, $m=\$18$, whence $x=4$ and $n=12$. But $y=n-3-x=5$. As $a=m/n=\$1.50$, $b=m/(n+3)=\$1.20$, and $c=\$1$, we readily find that
A owed \$4.50 for the first, and \$4.80 for the second month's service=\$9.30 total.
B owed \$6 for the second, and \$6 for the third month's service=\$12 total.
C owed \$7.50 for the first, and \$7 for the third month's service=\$14.50 total.

II. Solution by REV. J. H. MEYER, S. J., Professor of Mathematics, College of the Sacred Heart, Augusta, Ga.; by M. R. BECK, Cleveland, Ohio, and by J. EDWARD SANDERS, Reinersville, Ohio.

Using r as the number of dollars in one month's rent; x and y as the number of horses B and C put in at first, respectively, we obtain

$$\frac{rx}{3+x+y}=6, \text{ or } r=\frac{18+6x+6y}{x}. \qquad (1)$$

$$\frac{r(y+1)}{6+x+y}=7.20, \text{ or } r=\frac{43.2+7.2x+7.2y}{y+1}. \qquad (2)$$

$$\frac{5r}{9+x+y}=5, \text{ or } r=\frac{45+5x+5y}{5}. \qquad (3)$$

Equating (1) and (3), $90-15x+30y=5x^2+5xy$.........(4). Equating (2) and (3), $171+31x-14y=5xy+5y^2$..........(5). Adding (4) and (5), $-5(x+y)^2+16(x+y)=-261$..........(6). Solving (6) for $x+y$, $x+y=9$..........(7). Substituting (7) in (3), (2), (1), $r=18$, $y=5$, $x=4$. For first month, \$1.50, for the second, \$1.20, and for the third, \$1.00 must be paid for each horse.

Hence, A owed \$9.30; B, \$12.00; C, \$14.50.

Also solved by O. L. Callecot, A. H. Holmes, L. E. Newcomb, J. Scheffer, G. B. M. Zerr, and the Proposer.

257. Proposed by L. E. NEWCOMB, Los Gatos, Cal.

Solve (1) $x+y=10$, (2) $3x=\log_{10}y$.

I. Solution by HENRY HEATON, Belfield, N. D.

We have $3x=\log_{10}(10-x)$, or $10-x=10^{3x}$. If we suppose $x=\frac{1}{3}$, we obtain $9\frac{2}{3}=10$. An error of $\frac{1}{3}$. If we suppose $x=\frac{1}{4}$, we get $9.25=4.38$. An error of 4.87. By position we obtain $x=.328$. Substituting this we get $9.672=9.639$. An error of $-.043$. Substituting .329 for x we get $9.671=9.704$. An error of

.033. By position the second time we have $x=.3285$. Substituting this, we obtain $9.6715=9.6716$. An error of .0001.

Hence $x=.3285-$, and $y=10.-3285=9.6715+$.

II. Solution by A. H. HOLMES, Brunswick, Maine, and by J. SCHEFFER, Hagerstown, Md.

$x+y=10$ ....(1), $3x=\log_{10}y$ ....(2).

$\therefore \log y=\log(10-x)$ and $\log_{10}y=M\log(10-x)$. $\therefore \log(10-x)=3x/M$.

$$\log(10-x)=\frac{1}{M}+\log\left(1-\frac{x}{10}\right)=\frac{1}{M}-\frac{x}{10}-\frac{x^2}{200}-\frac{x^3}{3000}-\text{etc.}=\frac{3x}{M}.$$

The series converges very rapidly and the first two terms are sufficient for a correct value of x to the fourth decimal figure.

$$\text{Hence } \frac{1}{.434294}-\frac{x}{10}-\frac{x^2}{200}=\frac{3x}{.434294}.$$

$\therefore x=.3285-$, and from (1), $y=9.6714+$.

III. Solution by S. A. COREY, Hiteman, Iowa.

From (1), $y=10-x$, and substituting in (2), $\log_{10}(10-x)=3x$ ....(3), whence $\log_{10}[10(1-\frac{x}{10})]=3x$ ....(4), and $\log_{10}10+\log_{10}(1-\frac{x}{10})=3x$ ..... (5).

$\log_{10}(1-\frac{x}{10})=3x-1$ ....(6). But as $\log_{10}(1-\frac{x}{10})<0$, we have $3x<1$. Let $3x=1-3v$. By substituting in (6), $\log_{10}(\frac{29}{30}+\frac{v}{10})=-3v$, or $\log_{10}[\frac{29}{30}(1+\frac{3v}{29})]=-3v$, $\log_{10}(1+\frac{3v}{29})=\log_{10}30-\log_{10}29-3v=a-3v$ ($a=.014,723,256,8$) .... (7).

But $\log_{10}(1+\frac{3v}{29})>0$, hence $3v<a$. Next let $3v=a-3w$. By substituting in (7), $\log_{10}\left(\frac{29+a}{29}-\frac{3w}{29}\right)=3w$, $\log_{10}\left[\left(\frac{29+a}{29}\right)\left(1-\frac{3w}{29+a}\right)\right]=3w$. Whence $\log_{10}\left(1-\frac{3w}{29+a}\right)=3w+\log_{10}29-\log_{10}(29+a)=3w-.000,220,434,6$ ....(8).

But $\log_{10}\left(1-\frac{3w}{29+a}\right)=\frac{-3mw}{29+a}$ nearly, where m=modulus of common system of logarithms, and therefore (8) becomes $\frac{3mw}{29+a}+3w=.000,220,434,6$, nearly.

Whence, $w=.000,072,394,6$, nearly; and $x=\frac{1}{3}-a/3+w=.328,497,975,7$, $y=10-x=9.671,502,024,3$.

Also solved by G. W. Greenwood, G. B. M. Zerr, and the Proposer.